

HIDDEN QUANTUM-MECHANICAL SUPERSYMMETRY IN EXTRA DIMENSIONS*

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We study higher dimensional field theories with extra dimensions from a 4d spectrum point of view. It is shown that 4d mass spectra of spinor, gauge and gravity field theories are governed by quantum-mechanical supersymmetry. The 4d massless modes turn out to correspond to zero energy vacuum states of the supersymmetry. Allowed boundary conditions on extra dimensions compatible with the supersymmetry are found to be severely restricted.

Keywords: extra dimension; supersymmetry; 4d spectrum.

1. Introduction

Gauge theories in higher dimensions are a promising candidate beyond the Standard Model. Such theories turn out to possess unexpectedly rich properties that shed new light and give a deep understanding on high energy physics. In fact, it has been shown that new mechanisms of gauge symmetry breaking,¹⁻⁵ spontaneous supersymmetry breaking,⁶ and breaking of translational invariance^{7,8} can occur, and that various phase structures arise in field theoretical models on certain topological manifolds.⁹⁻¹¹ Furthermore, new diverse scenarios of solving the hierarchy problem have been proposed.¹²⁻¹⁵

Higher dimensional field theories will be described by 4d effective theories at low energies. Since we could not directly see extra dimensions, in particular, higher dimensional symmetries such as higher dimensional gauge symmetry and general covariance symmetry, one might ask what are remnants of the symmetries which originate from extra dimensions. They have

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to be hidden in the 4d effective theories. This is our motivation to investigate higher dimensional field theories from a 4d mass spectrum point of view. Our results show that the 4d mass spectrum is governed by quantum-mechanical supersymmetry (QM SUSY). Especially, the 4d massless spectrum is closely related to zero energy vacuum states of the supersymmetry and depends crucially on boundary conditions of extra dimensions, which are severely restricted by compatibility with QM SUSY.

In higher dimensional scalar theories, QM SUSY would appear in 4d spectrum but its appearance is found to be accidental. In higher dimensional spinor, gauge and gravity theories, QM SUSY always appears in 4d mass spectrum. Its origin turns out to be chiral symmetry, higher dimensional gauge symmetry and higher dimensional general covariance symmetry for spinor, gauge and gravity theories, respectively. It is interesting to note that all the symmetries guarantee the masslessness of the fields.

The paper is organized as follows: In section 2, we summarize the characteristic properties of QM SUSY. In the subsequent sections, we examine the 4d mass spectrum of higher dimensional scalar, spinor, gauge and gravity theories, separately and show that QM SUSY always appears in the 4d mass spectrum except for scalar theories. The section 7 is devoted to conclusions.

2. Minimal Supersymmetry Algebra

In any higher dimensional spinor/gauge/gravity theories with extra dimensions, quantum-mechanical supersymmetry turns out to be hidden in 4d spectrum and to play an important role to determine the spectrum of massless 4d fields, which are crucial ingredients in constructing low energy effective theories. The supersymmetric structure is found to be summarized in the minimal supersymmetry algebra, which consists of the hermitian operators H, Q and F , defined by

$$H = Q^2, \quad (1)$$

$$(-1)^F Q = -Q (-1)^F, \quad (2)$$

$$(-1)^F = \begin{cases} +1 & \text{for "bosonic" states,} \\ -1 & \text{for "fermionic" states,} \end{cases} \quad (3)$$

where H and Q are the Hamiltonian and the supercharge. The operator F is called a "fermion" number operator and the eigenvalues of $(-1)^F$ are given by $+1$ for "bosonic" states and -1 for "fermionic" ones, although the words, boson and fermion, have nothing to do with particles of integer spins

and half-odd integer spins. The readers should not confuse the quantum-mechanical supersymmetry with supersymmetry in quantum field theory, which implies a symmetry between bosonic states with integer spins and fermionic states with half-odd integer spins. The operators in the algebra (1)-(3) are defined in quantum-mechanical systems and hence the supercharge Q does not possess any spinor index. We call the symmetry which obeys the algebra (1)-(3) quantum-mechanical supersymmetry, or simply QM SUSY in this paper.

Let us first clarify characteristic properties of the algebra (1)-(3). We now show that if the system obeys the algebra, the spectrum has the following properties:

- 1) The energy eigenvalues are non-negative, i.e. $E \geq 0$.
- 2) Any positive energy state $|E, +\rangle$ of $(-1)^F = +1$ forms a pair with the state $|E, -\rangle$ of the same energy E and $(-1)^F = -1$, and vice versa. All positive energy states form supermultiplets.
- 3) Zero energy states (if exist) do not necessarily form pairs of supermultiplets.

Thus, a typical spectrum of QM SUSY systems will be given by Fig.1.

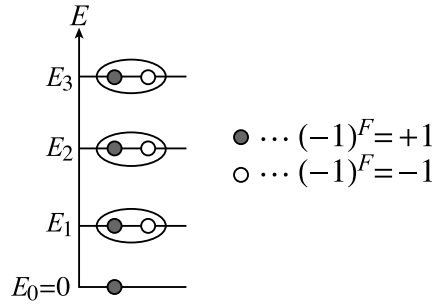


Fig. 1. A typical spectrum of QM SUSY

The first property 1) of $E \geq 0$ is derived from Eq.(1) because

$$E = \langle E|H|E\rangle = \langle E|Q^2|E\rangle = \|Q|E\rangle\|^2 \geq 0 \quad (4)$$

for any normalized energy eigenstate $|E\rangle$. Here, we have used the facts that the supercharge Q is hermitian and the norm of any state is non-negative. The second property 2) can be shown as follows: Suppose that $|E, +\rangle$ is an energy eigenstate with $(-1)^F = +1$. Then, the state $Q|E, +\rangle$ has the same energy eigenvalue E but the opposite eigenvalue of $(-1)^F$

because $H(Q|E, +) = Q^2(Q|E, +) = QH|E, +\rangle = E(Q|E, +)$ and $(-1)^F(Q|E, +) = -Q(-1)^F|E, +\rangle = -(Q|E, +)$. These relations imply that $Q|E, +\rangle \propto |E, -\rangle$. Assuming $Q|E, \pm\rangle = \alpha_{\pm}|E, \mp\rangle$ with $\| |E, \pm\rangle \|^2 = 1$, we find

$$E = \langle E, \pm | H | E, \pm \rangle = \langle E, \pm | Q^2 | E, \pm \rangle = \| Q | E, \pm \rangle \|^2 = |\alpha_{\pm}|^2. \quad (5)$$

We can then take $\alpha_{\pm} = \sqrt{E}$ without any loss of generality. Thus, the states $|E, +\rangle$ and $|E, -\rangle$ form a supermultiplet and are related each other through the SUSY relations

$$Q|E, \pm\rangle = \sqrt{E}|E, \mp\rangle. \quad (6)$$

The above result immediately shows that

$$Q|E, \pm\rangle = 0 \quad \text{for } E = 0. \quad (7)$$

This implies that zero energy states do not necessarily form supermultiplets.

We have found that characteristic properties of QM SUSY is nicely summarized in Fig.1. Thus, if we encounter such a spectrum, QM SUSY is expected to be hidden in the system. In fact, we will find this type of spectrum again and again in the following sections.

3. 5d Scalar

Let us start with a 5d real massless scalar field theory compactified on a circle S^1 .

$$S = \int d^4x \int_0^L dy \left\{ \frac{1}{2} \Phi(x, y) (\partial^\mu \partial_\mu + \partial_y^2) \Phi(x, y) - V(\Phi) \right\}, \quad (8)$$

where x^μ ($\mu = 0, 1, 2, 3$) denotes the 4-dimensional Minkowski space-time coordinate and y is the coordinate of the extra dimension on the circle S^1 of the circumference L . The $V(\Phi)$ denotes a potential term but it will not be concerned in our analysis.

Since the extra dimension is compactified on the circle of the circumference L , we have to specify a boundary condition on the field $\Phi(x, y)$. Let us take a periodic boundary condition, as an example, i.e.

$$\Phi(x, y + L) = \Phi(x, y). \quad (9)$$

We will make a comment on other boundary conditions at the end of this section.

In order to obtain the 4d mass spectrum, we expand the 5d field $\Phi(x, y)$ into the Kaluza-Klein modes such as

$$\Phi(x, y) = \phi_0^{(+)}(x)f_0^{(+)}(y) + \sum_{n=1}^{\infty} \{ \phi_n^{(+)}(x)f_n^{(+)}(y) + \phi_n^{(-)}(x)f_n^{(-)}(y) \}, \quad (10)$$

where $\phi_n^{(\pm)}(x)$ correspond to 4d scalar fields and $f_n^{(\pm)}(y)$ are the mass eigenfunctions of the differential operator $-\partial_y^2$, i.e.

$$\begin{cases} f_0^{(+)}(y) = N_0^{(+)}, \\ f_n^{(+)}(y) = N_n^{(+)} \cos\left(\frac{2\pi y}{L}\right), \\ f_n^{(-)}(y) = N_n^{(-)} \sin\left(\frac{2\pi y}{L}\right), \quad n = 1, 2, 3, \dots \end{cases} \quad (11)$$

Here, $N_n^{(\pm)}$ denote normalization constants. We should note that the set of $\{f_n^{(\pm)}\}$ forms a complete set so that the expansion (10) should be regarded as an identity.

Inserting the expansion (10) into the action (8) and using the orthogonal relations of $f_n^{(\pm)}(y)$ with an appropriate normalization, we find

$$\begin{aligned} S = \int d^4x \Big\{ & \frac{1}{2} \phi_0^{(+)}(x) \partial^\mu \partial_\mu \phi_0^{(+)}(x) + \frac{1}{2} \sum_{n=1}^{\infty} \left(\phi_n^{(+)}(x) (\partial^\mu \partial_\mu - m_n^2) \phi_n^{(+)}(x) \right. \\ & \left. + \frac{1}{2} \phi_n^{(-)}(x) (\partial^\mu \partial_\mu - m_n^2) \phi_n^{(-)}(x) \right) - V(\phi) \Big\}, \end{aligned} \quad (12)$$

where m_n is the 4d mass of the field $\phi_n^{(\pm)}$ and is given by

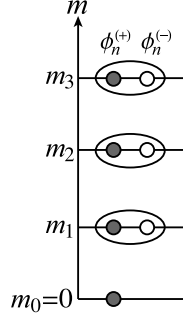
$$m_n = \frac{2\pi n}{L}, \quad n = 0, 1, 2, \dots \quad (13)$$

It follows that there appears a single massless mode $\phi_0^{(+)}$ and that all massive modes $\phi_n^{(\pm)}$ ($n = 1, 2, \dots$) are doubly degenerate. Thus, the 4d mass spectrum is given by Fig.2. This is nothing but a typical QM SUSY spectrum! We can, in fact, show that the minimal supersymmetry algebra appears in the system and the model will be the simplest higher dimensional field model that possesses QM SUSY.

Now the question is what are the operators H, Q and $(-1)^F$ of QM SUSY in the present system. The answer is

$$H = -\partial_y^2, \quad Q = -i\partial_y, \quad (-1)^F = \mathcal{P}. \quad (14)$$

The eigenvalues of the Hamiltonian H correspond to the mass squared m_n^2 . The supercharge Q is just the momentum operator, i.e. $Q = -i\partial_y$ and satisfies the desired relation $H = Q^2$. The operator $(-1)^F$ is given by the parity operator \mathcal{P} . It implies that the states of $(-1)^F = +1$ (-1)

Fig. 2. Spectrum of 5d scalar on $M^4 \times S^1$

correspond to even (odd) parity states. It is easy to verify that Q anticommutes with $(-1)^F$, as they should. The parity even (odd) function $f_n^{(+)}(y)$ ($f_n^{(-)}(y)$) has $(-1)^F = +1$ (-1) and they form a supermultiplet because $Qf_n^{(\pm)}(y) = -i\partial_y f_n^{(\pm)}(y) \propto f_n^{(\mp)}(y)$ for $n > 0$. Furthermore, $Qf_0^{(+)} = 0$ because $f_0^{(+)}$ is independent of y . This implies that the zero mode $f_0^{(+)}$ has no superpartner, as expected. Therefore, we have confirmed that the degeneracy for the nonzero modes in the 4d mass spectrum (see Fig.2) can be explained by QM SUSY.

As mentioned before, we make a comment on boundary conditions. In the above analysis, we assumed the periodic boundary condition (9). We can show that the system with the antiperiodic boundary condition $\Phi(y + L) = -\Phi(y)$ possesses QM SUSY as well but without any massless 4d state. Boundary conditions other than periodic and antiperiodic boundary conditions,^a however, lead to non-degenerate 4d spectrum and no QM SUSY.^b Thus, we conclude that QM SUSY found in the scalar field theory is accidental and there is no general mechanism to guarantee QM SUSY in any scalar field theories.

4. 5d Spinor

In this section, we consider a 5d spinor field on an interval ($0 \leq y \leq L$):

$$S = \int d^4x \int_0^L dy \bar{\Psi}(x, y) \left(i\gamma^\mu \partial_\mu + i\gamma^y \partial_y + M + \lambda\varphi(y) \right) \Psi(x, y), \quad (15)$$

^aExamples of other boundary conditions are Dirichlet boundary condition (b.c.) $\Phi(0) = 0$, Neumann b.c. $\partial_y \Phi(0) = 0$, twisted b.c. $\Phi(y + L) = e^{i\theta} \Phi(y)$ for a complex scalar.

^bThe systems with periodic or antiperiodic boundary condition can have an accidental symmetry, i.e. parity symmetry. This is the origin of QM SUSY as well as the degeneracy in the 4d spectrum.

where $\Psi(x, y)$ is a 4-component 5d Dirac spinor field and M is a 5-dimensional (bulk) mass. The γ^y is given by $\gamma^y \equiv \gamma^0 \gamma^1 \gamma^2 \gamma^3 = -i\gamma^5$. Here, we have introduced a coupling to a real scalar field $\varphi(y)$ and allow it to have a nontrivial y -dependence as a background field. The spinor $\Psi(x, y)$ can be expanded as

$$\Psi(x, y) = \sum_n \{ \psi_{+,n}(x) f_n(y) + \psi_{-,n}(x) g_n(y) \}, \quad (16)$$

where $\psi_{\pm,n}$ are 4-dimensional chiral spinors defined by $\gamma^5 \psi_{\pm,n} = \pm \psi_{\pm,n}$. The sets of functions $\{f_n(y)\}$ and $\{g_n(y)\}$ are assumed separately to form complete sets and should be chosen for $\psi_{\pm,n}(x)$ to be 4d mass eigenstates. From the representation theory of the Poincaré group, a massive 4d Dirac spinor ψ_n consists of chiral spinors $\psi_{+,n}$ and $\psi_{-,n}$ and they form the mass terms $m_n \bar{\psi}_{\pm,n} \psi_{\mp,n}$. On the other hand, a massless 4d spinor is chiral and hence does not necessarily form a pair of $\psi_{+,0}$ and $\psi_{-,0}$. Therefore, the 4d mass spectrum of (infinitely many) 4d spinors $\{\psi_{\pm,n}\}$ will be schematically given by Fig.3. This is nothing but a typical QM SUSY spectrum, as

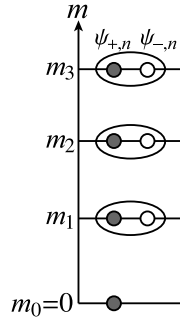


Fig. 3. Spectrum of 5d spinor

discussed in section 2. Thus, we expect that the minimal supersymmetry algebra is hidden in the 4d spectrum. This is indeed the case, as we will see below.

The 5d Dirac equation for $\Psi(x, y)$ is given by

$$[i\gamma^\mu \partial_\mu + \gamma^5 \partial_y + M + \lambda\varphi(y)] \Psi(x, y) = 0. \quad (17)$$

In terms of $\psi_{\pm,n}$, the above equation can be decomposed as

$$\sum_n (i\gamma^\mu \partial_\mu \psi_{+,n}(x)) f_n(y) + \sum_n \psi_{-,n}(x) (\mathcal{D}^\dagger g_n(y)) = 0, \quad (18)$$

$$\sum_n (i\gamma^\mu \partial_\mu \psi_{-,n}(x)) g_n(y) + \sum_n \psi_{+,n}(x) (\mathcal{D} f_n(y)) = 0, \quad (19)$$

where

$$\mathcal{D} = \partial_y + M + \lambda\varphi(y), \quad \mathcal{D}^\dagger = -\partial_y + M + \lambda\varphi(y). \quad (20)$$

We then require $f_n(y)$ and $g_n(y)$ to be the eigenfunctions of the differential operators $\mathcal{D}^\dagger \mathcal{D}$ and $\mathcal{D} \mathcal{D}^\dagger$, respectively, i.e.

$$\mathcal{D}^\dagger \mathcal{D} f_n(y) = m_n^2 f_n(y), \quad (21)$$

$$\mathcal{D} \mathcal{D}^\dagger g_n(y) = m_n^2 g_n(y). \quad (22)$$

Since $\mathcal{D}^\dagger \mathcal{D}$ and $\mathcal{D} \mathcal{D}^\dagger$ are hermitian,^c the sets of $\{f_n(y)\}$ and $\{g_n(y)\}$ form complete sets, as they should. It follows from Eqs.(21), (22) that $\mathcal{D} f_n$ ($\mathcal{D}^\dagger g_n$) obeys the same eigenequation (22) ((21)) as g_n (f_n), and hence that f_n and g_n are related each other through the SUSY relations

$$m_n g_n(y) = \mathcal{D} f_n(y), \quad (23)$$

$$m_n f_n(y) = \mathcal{D}^\dagger g_n(y), \quad (24)$$

with appropriate normalizations. Thus the eigenvalues of f_n and g_n are doubly degenerate (except for $m_n = 0$), as expected.

The minimal supersymmetry algebra is manifest by introducing the operators as

$$H = Q^2 = \begin{pmatrix} \mathcal{D}^\dagger \mathcal{D} & 0 \\ 0 & \mathcal{D} \mathcal{D}^\dagger \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & \mathcal{D}^\dagger \\ \mathcal{D} & 0 \end{pmatrix}, \quad (-1)^F = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (25)$$

Those operators act on 2-component wavefunctions

$$|\Psi\rangle = \begin{pmatrix} f(y) \\ g(y) \end{pmatrix}. \quad (26)$$

Now, it is not difficult to show that with the SUSY relations (23), (24) and with the orthonormal relations of $\{f_n(y)\}$ and $\{g_n(y)\}$ the action can be written, in terms of the 4d spinors, into the form

$$S = \int d^4x \{ \mathcal{L}_{m=0} + \mathcal{L}_{m \neq 0} \}, \quad (27)$$

^cBoundary conditions for $f_n(y)$ and $g_n(y)$ have to be chosen for \mathcal{D}^\dagger to be hermitian conjugate to \mathcal{D} . We will discuss how to determine boundary conditions later.

where $\mathcal{L}_{m=0}$ is the part of the Lagrangian consisting of massless chiral spinors and

$$\mathcal{L}_{m \neq 0} = \sum_{m_n \neq 0} \bar{\psi}_n(x) (i\gamma^\mu \partial_\mu + m_n) \psi_n(x) \quad (28)$$

with $\psi_n = \psi_{+,n} + \psi_{-,n}$ for $m_n \neq 0$. Thus, we have shown that $\psi_{\pm,n}$ are mass eigenstates with m_n , as announced before.

To determine the chiral zero mode part $\mathcal{L}_{m=0}$, we need to specify boundary conditions at $y = 0, L$ for $f_n(y)$ and $g_n(y)$. It turns out that the choice of boundary conditions is crucial for the existence of massless chiral spinors. Allowed boundary conditions compatible with QM SUSY have been classified and are listed below:^{16,17}

- i) $\mathcal{D}f_n(0) = 0 = \mathcal{D}f_n(L)$, $g_n(0) = 0 = g_n(L)$,
- ii) $f_n(0) = 0 = f_n(L)$, $\mathcal{D}^\dagger g_n(0) = 0 = \mathcal{D}^\dagger g_n(L)$,
- iii) $\mathcal{D}f_n(0) = 0 = f_n(L)$, $g_n(0) = 0 = \mathcal{D}^\dagger g_n(L)$,
- iv) $f_n(0) = 0 = \mathcal{D}f_n(L)$, $\mathcal{D}^\dagger g_n(0) = 0 = g_n(L)$.

Since the mode functions $f_n(y)$ and $g_n(y)$ obey the SUSY relations (23) and (24), chiral zero modes (if any) should satisfy

$$\mathcal{D}f_0(y) = 0, \quad (29)$$

$$\mathcal{D}^\dagger g_0(y) = 0, \quad (30)$$

with $m_0 = 0$. These first order differential equations can easily be solved as

$$f_0(y) = N_0 \exp \left\{ - \int_0^y dy' (M + \lambda \varphi(y')) \right\}, \quad (31)$$

$$g_0(y) = \bar{N}_0 \exp \left\{ + \int_0^y dy' (M + \lambda \varphi(y')) \right\}. \quad (32)$$

We should emphasize that the above solutions do not insure the existence of the massless chiral spinors $\psi_{+,0}$ and $\psi_{-,0}$ because they have to be discarded from the physical spectrum if $f_0(y)$ and/or $g_0(y)$ do not obey the boundary conditions. It is easy to see that $f_0(y)$ given in (31) obeys the boundary conditions only for i) and that $g_0(y)$ in (32) obeys them only for ii). Therefore, we find that

$$\mathcal{L}_{m=0} = \begin{cases} \bar{\psi}_{+,0}(x) i\gamma^\mu \partial_\mu \psi_{+,0}(x) & \text{for i),} \\ \bar{\psi}_{-,0}(x) i\gamma^\mu \partial_\mu \psi_{-,0}(x) & \text{for ii),} \\ 0 & \text{for iii) and iv).} \end{cases} \quad (33)$$

The extension of the above analysis to higher dimensions $M^4 \times K^N$ will be straightforward. The Γ -matrices on $M^4 \times K^N$ may be constructed, in

terms of the γ -matrices on M^4 and the $\bar{\gamma}$ -matrices on K^N , as

$$\begin{aligned}\Gamma^\mu &= \gamma^\mu \otimes I_{2[N/2]}, & \mu &= 0, 1, 2, 3, \\ \Gamma^i &= \gamma^5 \otimes \bar{\gamma}^i, & i &= 1, 2, \dots, N,\end{aligned}\quad (34)$$

which satisfy

$$\begin{aligned}\{\Gamma^\mu, \Gamma^\nu\} &= -2\eta^{\mu\nu} I_4 \otimes I_{2[N/2]}, & \mu, \nu &= 0, 1, 2, 3, \\ \{\Gamma^i, \Gamma^j\} &= -2\delta^{ij} I_4 \otimes I_{2[N/2]}, & i, j &= 1, 2, \dots, N, \\ \{\Gamma^\mu, \Gamma^j\} &= 0.\end{aligned}\quad (35)$$

Here, $[N/2]$ denotes the Gauss symbol and I_n is the $n \times n$ identity matrix. The structure of the Γ -matrices may imply that a $(4 + N)$ -dimensional spinor $\Psi(x, y)$ can be expanded as

$$\Psi(x, y) = \sum_n \{ \psi_{+,n}(x) \otimes \xi_{+,n}(y) + \psi_{-,n}(x) \otimes \xi_{-,n}(y) \}, \quad (36)$$

where $\psi_{\pm,n}(x)$ ($\xi_{\pm,n}(y)$) denote 4-dimensional (N -dimensional) spinors and x^μ (y^i) are the coordinates of M^4 (K^N). The 4d mass spectrum of $\psi_{\pm,n}$ will be schematically given just like Fig.3 and the mass eigenfunctions $\xi_{+,n}(y)$ and $\xi_{-,n}(y)$ will form a supermultiplet, though we will not proceed further.

5. 5d Vector

In this section, we consider a $(4+1)$ -dimensional abelian gauge theory on an interval ($0 \leq y \leq L$):

$$S = \int d^4x \int_0^L dy \sqrt{-g(y)} \left\{ -\frac{1}{4} F_{MN}(x, y) F^{MN}(x, y) \right\} \quad (37)$$

with a non-factorizable metric

$$ds^2 = e^{-4W(y)} \eta_{\mu\nu} dx^\mu dx^\nu + g_{55}(y) dy^2. \quad (38)$$

The metric reduces to the warped metric discussed by Randall and Sundrum¹² when $g_{55}(y) = 1$ and $W(y) = \frac{1}{2}k|y|$. Another choice of $g_{55}(y) = e^{-4W(y)}$ leads to the model discussed in Ref.[14], in which a hierarchical mass spectrum has been observed.

In order to expand the 5d gauge fields $A_\mu(x, y)$ and $A_y(x, y)$ into 4d mass eigenstates and to make a QM SUSY structure manifest, we introduce the

operators H, Q and $(-1)^F$ as follows:^{18,19}

$$H = Q^2 = \begin{pmatrix} -\frac{1}{\sqrt{g_{55}}} \partial_y \frac{e^{-4W}}{\sqrt{g_{55}}} \partial_y & 0 \\ 0 & -\partial_y \frac{1}{\sqrt{g_{55}}} \partial_y \frac{e^{-4W}}{\sqrt{g_{55}}} \end{pmatrix}, \quad (39)$$

$$Q = \begin{pmatrix} 0 & -\frac{1}{\sqrt{g_{55}}} \partial_y \frac{e^{-4W}}{\sqrt{g_{55}}} \\ \partial_y & 0 \end{pmatrix}, \quad (40)$$

$$(-1)^F = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (41)$$

which act on two-component vectors

$$|\Psi\rangle = \begin{pmatrix} f(y) \\ g(y) \end{pmatrix}. \quad (42)$$

The inner product of two states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ is defined by

$$\langle \Psi_2 | \Psi_1 \rangle = \int_0^L dy \sqrt{g_{55}(y)} \left\{ f_2(y) f_1(y) + \frac{e^{-4W(y)}}{g_{55}(y)} g_2(y) g_1(y) \right\}. \quad (43)$$

To obtain consistent boundary conditions for the functions $f(y)$ and $g(y)$ in $|\Psi\rangle$, we first require that the supercharge Q is hermitian with respect to the inner product (43), i.e.

$$\langle \Psi_2 | Q \Psi_1 \rangle = \langle Q \Psi_2 | \Psi_1 \rangle. \quad (44)$$

It turns out that the functions $f(y)$ and $g(y)$ have to obey one of the following four types of boundary conditions:

$$\text{i) } g(0) = g(L) = 0, \quad (45)$$

$$\text{ii) } f(0) = f(L) = 0, \quad (46)$$

$$\text{iii) } g(0) = f(L) = 0, \quad (47)$$

$$\text{iv) } f(0) = g(L) = 0. \quad (48)$$

We further require that the state $Q|\Psi\rangle$ obeys the same boundary conditions as $|\Psi\rangle$, otherwise Q is not a well defined operator and “bosonic” and “fermionic” states would not form supermultiplets. The requirement leads

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to

$$\partial_y f(0) = \partial_y f(L) = 0 \quad \text{for i),} \quad (49)$$

$$\partial_y \left(\frac{e^{-4W}}{\sqrt{g_{55}}} g \right) (0) = \partial_y \left(\frac{e^{-4W}}{\sqrt{g_{55}}} g \right) (L) = 0 \quad \text{for ii),} \quad (50)$$

$$\partial_y f(0) = \partial_y \left(\frac{e^{-4W}}{\sqrt{g_{55}}} g \right) (L) = 0 \quad \text{for iii),} \quad (51)$$

$$\partial_y \left(\frac{e^{-4W}}{\sqrt{g_{55}}} g \right) (0) = \partial_y f(L) = 0 \quad \text{for iv).} \quad (52)$$

Combining all the above results, we have found the four types of boundary conditions compatible with supersymmetry,^{18,19}

$$\text{Type (N, N) : } \begin{cases} \partial_y f(0) = \partial_y f(L) = 0, \\ g(0) = g(L) = 0, \end{cases} \quad (53)$$

$$\text{Type (D, D) : } \begin{cases} f(0) = f(L) = 0, \\ \partial_y \left(\frac{e^{-4W}}{\sqrt{g_{55}}} g \right) (0) = \partial_y \left(\frac{e^{-4W}}{\sqrt{g_{55}}} g \right) (L) = 0, \end{cases} \quad (54)$$

$$\text{Type (N, D) : } \begin{cases} \partial_y f(0) = f(L) = 0, \\ g(0) = \partial_y \left(\frac{e^{-4W}}{\sqrt{g_{55}}} g \right) (L) = 0, \end{cases} \quad (55)$$

$$\text{Type (D, N) : } \begin{cases} f(0) = \partial_y f(L) = 0, \\ \partial_y \left(\frac{e^{-4W}}{\sqrt{g_{55}}} g \right) (0) = g(L) = 0. \end{cases} \quad (56)$$

It follows that the above boundary conditions ensure the hermiticity of the Hamiltonian, i.e.

$$\langle \Psi_2 | H \Psi_1 \rangle = \langle H \Psi_2 | \Psi_1 \rangle. \quad (57)$$

Therefore, we have succeeded to obtain the consistent set of boundary conditions that ensure the hermiticity of the supercharge and the Hamiltonian and also that the action of the supercharge on $|\Psi\rangle$ is well defined. Since the supersymmetry is a direct consequence of higher-dimensional gauge invariance, our requirements on boundary conditions should be, at least, necessary conditions to preserve it. It turns out that the boundary conditions obtained above are consistent with those in Ref.[5], although it is less obvious how the requirement of the least action principle proposed in Ref.[5] is connected to gauge invariance. We should emphasize that the supercharge Q is well defined for all the boundary conditions (53)-(56) and hence that the supersymmetric structure always appears in the spectrum, though the boundary conditions other than the type (N,N) break 4d gauge symmetries, as we will see below.

From the above analysis, the 5d gauge fields $A_\mu(x, y)$ and $A_y(x, y)$ are expanded in the mass eigenstates as follows:

$$A_\mu(x, y) = \sum_n A_{\mu,n}(x) f_n(y), \quad (58)$$

$$A_y(x, y) = \sum_n h_n(x) g_n(y), \quad (59)$$

where $f_n(y)$ and $g_n(y)$ are the eigenstates of the Schrödinger-like equations

$$-\frac{1}{\sqrt{g_{55}}} \partial_y \frac{e^{-4W}}{\sqrt{g_{55}}} \partial_y f_n(y) = m_n^2 f_n(y), \quad (60)$$

$$-\partial_y \frac{1}{\sqrt{g_{55}}} \partial_y \frac{e^{-4W}}{\sqrt{g_{55}}} g_n(y) = m_n^2 g_n(y) \quad (61)$$

with one of the four types of the boundary conditions (53)-(56) and they are actually related each other through the SUSY relations:

$$m_n g_n(y) = \partial_y f_n(y), \quad (62)$$

$$m_n f_n(y) = -\frac{1}{\sqrt{g_{55}}} \partial_y \frac{e^{-4W}}{\sqrt{g_{55}}} g_n. \quad (63)$$

Since the massless states are especially important in phenomenology, let us investigate the massless states of the equations (60) and (61). Thanks to supersymmetry, the massless modes would be the solutions to the first order differential equation $Q|\Psi_0\rangle = 0$, i.e.

$$\partial_y f_0(y) = 0, \quad (64)$$

$$\partial_y \left(\frac{e^{-4W}}{\sqrt{g_{55}}} g_0(y) \right) = 0. \quad (65)$$

The solutions are easily found to be

$$f_0(y) = N_0, \quad (66)$$

$$g_0(y) = \bar{N}_0 e^{4W(y)} \sqrt{g_{55}(y)}, \quad (67)$$

where N_0 and \bar{N}_0 are some constants. We should emphasize that the above solutions do not necessarily imply physical massless states of $A_{\mu,0}(x)$ and $h_0(x)$ in the spectrum. This is because the boundary conditions exclude some or all of them from the physical spectrum. Indeed, $f_0(y)$ ($g_0(y)$) satisfies only the boundary conditions of the type (N,N) (type (D,D)). Thus, a massless vector $A_{\mu,0}(x)$ (a massless scalar $h_0(x)$) appears only for the type (N,N) (type (D,D)) boundary conditions (see Fig.4). This implies that the 4d gauge symmetry is broken except for the type (N,N) boundary conditions.

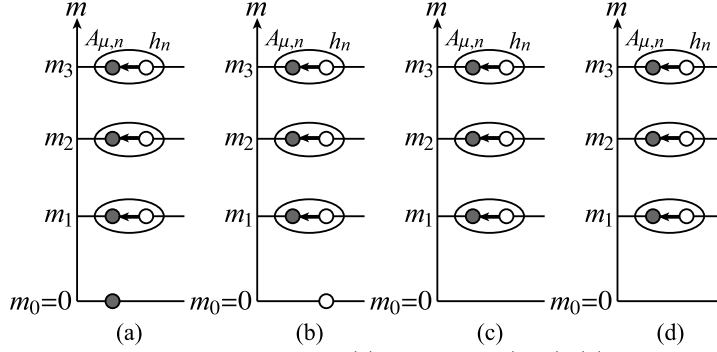


Fig. 4. Spectrum of the 5d gauge theory (a) for the type (N,N), (b) for the type (D,D), (c) for the type (N,D), (d) for the type (D,N) boundary conditions.

It is instructive to discuss the relation between the QM SUSY and the higher dimensional gauge symmetry. The relation becomes apparent by expressing the action, in terms of the 4d mass eigenstates, as

$$S = \int d^4x \{ \mathcal{L}_{m=0} + \mathcal{L}_{m \neq 0} \}, \quad (68)$$

where $\mathcal{L}_{m=0}$ is the Lagrangian consisting of the massless fields and

$$\mathcal{L}_{m \neq 0} = \sum_{m_n \neq 0} \left\{ -\frac{1}{4} (F_{\mu\nu,n}(x))^2 - \frac{m_n^2}{2} \left(A_{\mu,n}(x) - \frac{1}{m_n} \partial_\mu h_n(x) \right)^2 \right\} \quad (69)$$

with $F_{\mu\nu,n} = \partial_\mu A_{\nu,n} - \partial_\nu A_{\mu,n}$. It follows that every nonzero mode $h_n(x)$ for $m_n \neq 0$ can be absorbed into the longitudinal mode of $A_{\mu,n}(x)$ and then $A_{\mu,n}(x)$ becomes massive with three degrees of freedom, as it should be. The choice of $h_n(x) = 0$ ($m_n \neq 0$) is called a unitary gauge. It should be emphasized that the Lagrangian (69) has been derived by use of the SUSY relations (62) and (63). Therefore, the QM SUSY is necessary for $A_{\mu,n}(x)$ ($m_n \neq 0$) to become massive by absorbing the unphysical mode $h_n(x)$, which is a consequence of the higher dimensional gauge symmetry. This observation is summarized in Fig.4.

We have restricted our considerations to a 5d gauge theory. The extension to any higher dimensional gauge theory is possible and QM SUSY is found in the 4d mass spectrum. The details have been given in Ref.[18, 19].

6. 5d Gravity

In this section, we investigate the 4d mass spectrum of the 5d Randall-Sundrum gravity theory¹² with a warped metric

$$ds^2 = e^{2A(y)} (\eta_{\mu\nu} dx^\mu dx^\nu + dy^2), \quad (70)$$

where $A(y)$ is the warp factor which turns out to play a role of a superpotential in the $N = 2$ Witten model.²⁰ For the Randall-Sundrum model, the warp factor is given by

$$A(y) = -\ln\left(\frac{y}{y_1}\right). \quad (71)$$

Here, the location of the UV brane is chosen such that the warp factor is set equal to 1 on the UV brane at $y = y_1$.

The metric fluctuations h_{MN} around the background metric (70) are given by

$$ds^2 = e^{2A(y)} (\eta_{MN} + h_{MN}(x, y)) dx^M dx^N \quad (72)$$

and h_{MN} turn out to be useful with the parameterization²¹

$$h_{MN}(x, y) = \begin{pmatrix} h_{\mu\nu}(x, y) - \frac{1}{2}\eta_{\mu\nu}\phi(x, y) & h_{y\nu}(x, y) \\ h_{\mu y}(x, y) & \phi(x, y) \end{pmatrix}. \quad (73)$$

The action is invariant under infinitesimal general coordinate transformations: $x^M \rightarrow x'^M = x^M + \xi^M(x, y)$, which are translated into the field transformations of the metric fluctuations:

$$\begin{aligned} \delta h_{\mu\nu} &= -\partial_\mu \xi_\nu - \partial_\nu \xi_\mu - \eta_{\mu\nu} (\partial_y + 3A') \xi_y, \\ \delta h_{\mu y} &= -\partial_y \xi_\mu - \partial_\mu \xi_y, \\ \delta \phi &= -2(\partial_y + A') \xi_y, \end{aligned} \quad (74)$$

where $A'(y) = dA(y)/dy$. The metric fluctuation fields are expanded, in terms of some complete sets of the functions $\{f_n(y)\}$, $\{g_n(y)\}$, $\{k_n(y)\}$, as

$$\begin{aligned} h_{\mu\nu}(x, y) &= \sum_n h_{\mu\nu,n}(x) f_n(y), \\ h_{\mu y}(x, y) &= \sum_n h_{\mu y,n}(x) g_n(y), \\ \phi(x, y) &= \sum_n \phi_n(x) k_n(y). \end{aligned} \quad (75)$$

It is instructive to examine degrees of freedom for massive modes. Each field of $h_{\mu\nu,n}$, $h_{\mu y,n}$, ϕ has originally 2, 2, 1 degrees of freedom, respectively because 5d gravity has no mass term in a 5-dimensional point of view. The

vector field $h_{\mu y, n}$ could become massive by “eating” one extra degree of freedom and then has three degrees of freedom as a massive vector. The tensor field $h_{\mu\nu, n}$ could become massive by “eating” three extra degrees of freedom and then has five degrees of freedom as a massive graviton. The above Higgs-like mechanism can actually occur in the 5d gravity system. The vector field $h_{\mu y, n}$ “eats” ϕ_n to become a massive vector with three degrees of freedom, and then the tensor field $h_{\mu\nu, n}$ “eats” $h_{\mu y, n}$ to become a massive graviton with five degrees of freedom:

$$\overbrace{h_{\mu\nu, n} \quad \underbrace{h_{\mu y, n} \quad \phi_n}_{\text{a massive vector}}}^{\text{a massive graviton}} \quad (76)$$

The above observation strongly suggests, on the analogy of the 5d gauge theory, that the 5d gravity theory possesses *two* QM SUSY systems in the 4d spectrum: One is realized between the eigenfunctions $g_n(y)$ and $k_n(y)$. The other is between $f_n(y)$ and $g_n(y)$. To verify it, we have to find the eigenequations for $f_n(y), g_n(y)$ and $k_n(y)$, which should diagonalize the quadratic action for $h_{\mu\nu, n}(x), h_{\mu y, n}(x)$ and $\phi_n(x)$. The eigenequations for $f_n(y), g_n(y)$ and $k_n(y)$ are found to be²¹

$$-(\partial_y^2 + 3A'(y)\partial_y)f_n(y) = m_n^2 f_n(y), \quad (77)$$

$$-(\partial_y^2 + 3A'(y)\partial_y + 3A''(y))g_n(y) = m_n^2 g_n(y), \quad (78)$$

$$-(\partial_y^2 + 3A'(y)\partial_y + 4A''(y))k_n(y) = m_n^2 k_n(y). \quad (79)$$

The supersymmetric structure between $f_n(y)$ and $g_n(y)$ will become apparent if we express Eqs.(77) and (78) into the form

$$\mathcal{D}^\dagger \mathcal{D} f_n(y) = m_n^2 f_n(y), \quad \mathcal{D} \mathcal{D}^\dagger g_n(y) = m_n^2 g_n(y), \quad (80)$$

where

$$\mathcal{D} = \partial_y, \quad \mathcal{D}^\dagger = -(\partial_y + 3A'(y)). \quad (81)$$

The eigenfunctions $f_n(y)$ and $g_n(y)$ are actually related each other through the SUSY relations

$$m_n g_n(y) = \mathcal{D} f_n(y), \quad m_n f_n(y) = \mathcal{D}^\dagger g_n(y). \quad (82)$$

It seems strange that \mathcal{D}^\dagger is hermitian conjugate to \mathcal{D} . This is, however, true because the inner product is defined by²¹

$$\langle \psi | \phi \rangle = \int_{y_2}^{y_1} dy e^{3A(y)} (\psi(y))^* \phi(y) \quad (83)$$

with the boundary conditions

$$\partial_y f_n(y) = 0 = g_n(y), \quad \text{at } y = y_1, y_2. \quad (84)$$

The factor $e^{3A(y)}$ in Eq.(83) is required because of the presence of it in the action, whose origin comes from the nontrivial background metric (70). The boundary conditions (84) turn out to be compatible with supersymmetry. The QM SUSY structure will be manifest if we introduce two component wavefunctions

$$|\Psi\rangle = \begin{pmatrix} f(y) \\ g(y) \end{pmatrix}. \quad (85)$$

Then, H, Q and $(-1)^F$ are found to be the same form as Eq.(25) with \mathcal{D} and \mathcal{D}^\dagger defined in Eqs.(81).

Let us next proceed to the analysis of a pair of the eigenfunctions $g_n(y)$ and $k_n(y)$. The supersymmetric structure between them will be apparent if we express Eqs.(78), (79) into the form²¹

$$\bar{\mathcal{D}}^\dagger \bar{\mathcal{D}} g_n(y) = m_n^2 g_n(y), \quad \bar{\mathcal{D}} \bar{\mathcal{D}}^\dagger k_n(y) = m_n^2 k_n(y), \quad (86)$$

where

$$\bar{\mathcal{D}} = \partial_y + A'(y), \quad \bar{\mathcal{D}}^\dagger = -(\partial_y + 2A'(y)). \quad (87)$$

Here, we have used the relation $(A')^2 = A''$. The eigenfunctions $g_n(y)$ and $k_n(y)$ are related through the SUSY relations

$$m_n k_n(y) = \bar{\mathcal{D}} g_n(y), \quad m_n g_n(y) = \bar{\mathcal{D}}^\dagger k_n(y). \quad (88)$$

The inner product is defined by Eq.(83). This guarantees that $\bar{\mathcal{D}}$ and $\bar{\mathcal{D}}^\dagger$ are hermitian conjugate each other with the boundary conditions

$$g_n(y) = 0 = \bar{\mathcal{D}}^\dagger k_n(y), \quad \text{at } y = y_1, y_2. \quad (89)$$

The supersymmetric structure is manifest if we introduce two component wavefunctions in a similar manner as Eq.(85). Then, \bar{H}, \bar{Q} and $(-1)^{\bar{F}}$ are given by the same form as Eq.(25) with the replacement of \mathcal{D} and \mathcal{D}^\dagger by Eqs.(87).

We have found two QM SUSY systems, as expected. The eigenfunctions $f_n(y)$ and $g_n(y)$ form a supermultiplet, and $g_n(y)$ and $k_n(y)$ form another supermultiplet. The QM SUSY structure turns out to severely restrict the allowed boundary conditions for $f_n(y), g_n(y)$ and $k_n(y)$. In fact, the boundary conditions for them are unique in order to be compatible with the QM SUSY. This fact is especially important in a low energy effective theory point of view. This is because the boundary conditions for $f_n(y), g_n(y)$ and

$k_n(y)$ with the eigenvalue equations (80) and (86) determine uniquely the massless modes. In the present Randall-Sundrum model, there exist one massless graviton and one massless scalar (radion). The 4d spectrum of the Randall-Sundrum model is depicted in Fig.5.

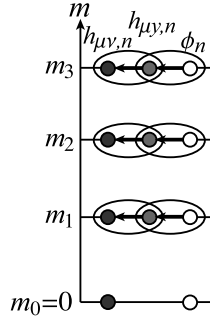


Fig. 5. Spectrum of the 5d Randall-Sundrum gravity

We have investigated the 5d Randall-Sundrum model in which the 4-dimensional space-time is a flat Minkowski. Karch and Randall²² have extended it to 4d de Sitter (dS_4) and anti de Sitter (AdS_4) space-time. In those cases, the warp factor $A(y)$ is different from Eq. (71) with a non-vanishing 4d cosmological constant. The analysis proceeds in a similar way and the results will be reported elsewhere.

Finally, we should make a few comments on interesting observations. The warp factor $A(y)$ cannot be an arbitrary function but has to be a solution of the Einstein equation. This gives a non-trivial constraints on $A(y)$. The differential equations for $f_n(y)$, $g_n(y)$ and $k_n(y)$ are found to be in a class of exactly solvable models with the property of shape invariance.²³ This property holds even for the Karch-Randall models. The second interesting observation is the uniqueness of the boundary conditions which have to be compatible with two QM SUSYs. If we would have a 5d massless theory with a higher spin ($s > 2\hbar$), the 4d mass spectrum of the system could possess more than three QM SUSYs. Our analysis, however, tells us that there are no possible boundary conditions compatible with all QM SUSYs. This may lead to a conclusion that any 5d massless theory on an interval with higher spins ($s > 2\hbar$) has no possible boundary conditions compatible with QM SUSYs. This seems to be consistent with the fact that any non-trivial massless higher spin theory with $s > 2\hbar$ has not been found yet.

7. Conclusions

We have investigated higher dimensional scalar, spinor, gauge and gravity theories from a 4d spectrum point of view. Our analysis has shown that QM SUSY is hidden in 4d mass spectrum of any higher dimensional field theories except for scalars. The origins of QM SUSYs in the 4d mass spectrum are found to be chiral symmetry, higher dimensional gauge symmetry and higher dimensional general covariance symmetry for spinor, gauge and gravity theories, respectively. There is no such symmetry to guarantee QM SUSY in the 4d mass spectrum for scalar theories. QM SUSY could appear in higher dimensional scalar theories but it is an accidental symmetry.

The higher dimensional gauge invariance guarantees that the nonzero vector mode $A_{\mu,n}(x)$ ($n > 0$) can absorb the unphysical scalar mode $h_n(x)$ to become massive with three degrees of freedom. This is the origin of QM SUSY between the mass eigenfunctions $f_n(y)$ and $g_n(y)$ for $A_{\mu,n}(x)$ and $h_n(x)$.

The higher dimensional general covariance symmetry similarly guarantees that the nonzero vector mode $h_{\mu y,n}(x)$ ($n > 0$) can absorb the unphysical scalar mode $\phi_n(x)$ to become massive with three degrees of freedom, and that the nonzero graviton mode $h_{\mu\nu,n}(x)$ ($n > 0$) can then absorb the massive mode $h_{\mu y,n}(x)$ to become massive with five degrees of freedom. This is the origin of QM SUSY and there appear *two* QM SUSYs in higher dimensional gravity theories: One connects the mass eigenfunction $g_n(y)$ to $k_n(y)$. The other connects the mass eigenfunction $f_n(y)$ to $g_n(y)$.

It is interesting to point out that the chiral, higher dimensional gauge and higher dimensional general covariance symmetries are all related to the symmetries that guarantee the masslessness of spinor, vector and tensor fields, respectively. Since massless particles are crucially important at low energy physics, it would be of great interest to investigate QM SUSY in more details. Our results will be summarized in the Table 1.

Table 1. Summary of our results

higher dim. fields	QM SUSY	origin
scalar	\triangle	accidental
spinor	\bigcirc	chiral symmetry
vector	\bigcirc	higher dim. gauge symmetry
tensor	\bigcirc	higher dim. general covariance symmetry

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